MBF 3C Unit 7 (Exponential Relations) Outline

Day	Lesson Title	Specific Expectations
1	Negative and Zero Exponents	A2.1, A2.2
2	Exponent Rules	A2.3
3	Graph Exponential Relations	A2.4, A2.5, A2.6
4	Characteristics of Exponential Relations	A2.5
5, 6, 7, 8	Applications of Exponential Relations	A3.1, A3.2, A3.3, A3.4
9	Review Day	
10	Test Day	
TOTAL DAYS:		10

A2.1 – determine, through investigation using a variety of tools and strategies (e.g., graphing with technology; looking for patterns in tables of values), and describe the meaning of negative exponents and of zero as an exponent;

A2.2 – evaluate,with and without technology, numerical expressions containing integer exponents and rational bases (e.g., 2–3, 63, 34560, 1.0310);

A2.3 – determine, through investigation (e.g., by patterning with and without a calculator), the exponent rules for multiplying and dividing numerical expressions involving exponents [e.g., $\left(\frac{1}{2}\right)^3 \times \left(\frac{1}{2}\right)^2$ and the exponent rule for simplifying numerical expressions involving a power of a power [e.g., $(53)^2$];

A2.4 – graph simple exponential relations, using paper and pencil, given their equations [e.g., $y = 2x$, $y = 10x$, $y = (1/2)^x$];

A2.5 – make and describe connections between representations of an exponential relation (i.e., numeric in a table of values; graphical; algebraic);

A2.6 – distinguish exponential relations from linear and quadratic relations by making comparisons in a variety of ways (e.g., comparing rates of change using finite differences in tables of values; inspecting graphs; comparing equations), within the same context when possible (e.g., simple interest and compound interest; population growth) (Sample problem: Explain in a variety of ways how you can distinguish exponential growth represented by $y = 2x$ from quadratic growth represented by $y = x^2$ and linear growth represented by $y = 2x$.).

A3.1 – collect data that can be modelled as an exponential relation, through investigation with and without technology, from primary sources, using a variety of tools (e.g., concrete materials such as number cubes, coins; measurement tools such as electronic probes), or from secondary sources (e.g.,websites such as Statistics Canada, E-STAT), and graph the data (Sample problem: Collect data and graph the cooling curve representing the relationship between temperature and time for hot water

cooling in a porcelain mug. Predict the shape of the cooling curve when hot water cools in an insulated mug.Test your prediction.);

A3.2 – describe some characteristics of exponential relations arising from real-world applications (e.g., bacterial growth, drug absorption) by using tables of values (e.g., to show a constant ratio, or multiplicative growth or decay) and graphs (e.g., to show, with technology, that there is no maximum or minimum value);

A3.3 – pose and solve problems involving exponential relations arising from a variety of real-world applications (e.g., population growth, radioactive decay, compound interest) by using a given graph or a graph generated with technology from a given equation (Sample problem: Given a graph of the population of a bacterial colony versus time, determine the change in population in the first hour.);

A3.4 – solve problems using given equations of exponential relations arising from a variety of real-world applications (e.g., radioactive decay, population growth, height of a bouncing ball, compound interest) by substituting values for the exponents into the equations (Sample problem: The height, h metres, of a ball after n bounces is given by the equation $h = 2(0.6)^n$. Determine the height of the ball after 3 bounces.

Unit 7 Day 1: Exponential Relations			MBF 3C
	Description Determine exponent rules		Materials Coloured markers Calculators BLM 7.1.1 BLM 7.1.2
			Assessment Opportunities
Minds On…	**Whole Class →Discussion** Discuss previous lesson's key concepts, and journal entries Demonstrate how to multiply: $4^2 \times 4^4 = 16 \times 64 = 1024$		
Action!	**Small Groups →Investigation** Small groups of students use calculators and "Exponents I" activity, BLM 7.1.1, to investigate exponent rules. Students work individually on their sections (Partner A completes 1a, 2a, and 3a)		
Consolidate Debrief	**Whole Class →Presentations** One member from each group explains what their group accomplished, and summarizes findings. Encourage students to question each other and give constructive comments Encourage students to add exponent rules to their unit cover page.		
Reflection *Skill Drill*	**Home Activity or Further Classroom Consolidation** Journal entry: "Write a letter to your teacher explaining what you understand about exponents and what needs to be clarified." BLM 7.1.2		

MBF3C
BLM 7.1.1

Name:
Date:

Exponents Take I

In your groups of 3, label yourselves Partner A, Partner B, and Partner C.

- Each partner completes their respective section of the investigation.
- When your group is finished, check your answers and complete Part B.
- Each group member should be prepared to speak about their findings.

Part A

For question 1-3, Partner A completes (a), Partner B completes (b), and Partner C completes (c). **Keep fractions in fraction form.**

1.

a. $3^2 \times 3^3$	_____	$(\frac{1}{2})^2 \times (\frac{1}{2})^3$=	_____	$(.1)^5 \times (.1)^3$=	_____
=					
3^5 =	_____	$(\frac{1}{2})^5$=	_____	$(.1)^8$=	_____
b. $2^3 \times 2^5$=	_____	$(\frac{1}{3})^2 \times (\frac{1}{3})^1$=	_____	$(.2)^3 \times (.2)^4$=	_____
2^8 =	_____	$(\frac{1}{3})^3$=	_____	$(.2)^7$=	_____
c. $5^2 \times 5^4$=	_____	$(\frac{1}{4})^3 \times (\frac{1}{4})^2$=	_____	$(.3)^1 \times (.3)^5$=	_____
5^6 =	_____	$(\frac{1}{4})^5$=	_____	$(.3)^6$=	_____

Describe what you notice.

2. Fill in the following table.

a. $2^8 \div 2^5$=	_____	$(\frac{1}{4})^3 \div (\frac{1}{4})^1$=	_____	$(.2)^7 \div (.2)^3$=	_____
2^3 =	_____	$(\frac{1}{4})^2$=	_____	$(.2)^4$=	_____
b. $5^6 \div 5^2$=	_____	$(\frac{1}{2})^5 \div (\frac{1}{2})^3$=	_____	$(.3)^6 \div (.3)^5$=	_____
5^4 =	_____	$(\frac{1}{2})^2$=	_____	$(.3)^1$=	_____
c. $3^5 \div 3^2$=	_____	$(\frac{1}{3})^5 \div (\frac{1}{3})^3$=	_____	$(.1)^8 \div (.1)^5$=	_____
3^3 =	_____	$(\frac{1}{3})^2$=	_____	$(.1)^3$=	_____

Describe what you notice**.**

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BLM 7.1.1

Name:
Date:

Exponents Take I (Continued)

3.

a. $(2^3)^2=$ _____	$(.3^2)^2=$ _____	$(\frac{1}{2}^4)^1=$ _____
$2^6=$ _____	$.3^4=$ _____	$(\frac{1}{2})^4=$ _____
b. $(3^2)^3=$ _____	$(5^3)^2=$ _____	$(\frac{1}{3}^4)^2=$ _____
$3^6=$ _____	$5^6=$ _____	$(\frac{1}{3})^8=$ _____
c. $(.1^3)^2=$ _____	$(2^2)^4=$ _____	$(\frac{1}{4}^2)^3=$ _____
$.1^6=$ _____	$2^8=$ _____	$(\frac{1}{4})^6=$ _____

Describe what you notice.

Part B

1. As a group, summarize your findings:

2. Other important points:

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BLM 7.1.2

Exponents

Name:
Date:

Evaluate. Rewrite negative exponents and evaluate as fractions.

1) $5^4 \times 5^2$	2) $2^4 \times 2^{-2}$	3) $3^{-3} \times 2^{-2}$	4) $8^{-3} \times 8^5$
5) $4^3 \times 4^{-2}$	6) $6^2 \times 6^4$	7) $7^{-3} \times 7^6$	8) $5^4 \times 5^{-2}$
9) $4^4 \div 4^2$	10) $3^2 \div 3^5$	11) $2 \div 2^{-2}$	12) $5^{-2} \div 5^{-2}$
13) $7^3 \div 7^2$	14) $10^7 \div 10^4$	15) $6^4 \div 6^{-3}$	16) $8^{-2} \div 8^{-1}$
17) $(2^3)^4$	18) $(3^2)^3$	19) $(4^1)^5$	20) $(5^{-2})^3$
21) $(4^2)^{-3}$	22) $(2^{-3})^{-4}$	23) $(3^{-2})^{-3}$	24) $(4^{-1})^{-5}$
25) $(2^{-3})^3$	26) $3^a \times 3^b \times 3^c = 3^?$		27) $4^{a+b} \times 4^{a-b} = 4^?$

Unit 7 Day 2: Exponential Relations			MBF 3C
	Description Determine and describe the meaning of negative and zero exponents, and evaluate with and without technology		Materials BLM 7.2.1 BLM 7.2.2 Calculators BLM 7.2.3
			Assessment Opportunities
Minds On…	**Whole Class →Discussion** Review meaning of exponent • What is an exponent? • How are exponents used? • Why are exponents used? Students create a cover/title page on blank paper for the unit on Exponential Relations to be placed in their binders/notes.		If not using the Unit Cover Page concept for binders, use a wall poster that can be added to after each lesson.
Action!	**Whole Class →Teacher Directed Guided Discovery** Teacher leads the class in a guided discovery of the exponent laws for zero and negative exponents. "Exponents I" activity, BLM 7.1.1 **Whole Class → Discussion** Discuss findings of pairs investigation. Students suggest rules for negative and zero exponents. **Pairs →Investigation** Pairs of students complete "Exponents II" activity, BLM 7.1.2, *with* a calculator. Encourage checking among groups once students appear finished.		Students may need to be reminded that dividing by 2, or 3, or 5 is the same as multiplying by $\frac{1}{2}$, or $\frac{1}{3}$ or $\frac{1}{5}$
Consolidate Debrief	**Whole Class →Pairs Summaries** Encourage communication using oral summaries of key concepts. Encourage students to put rules on their unit cover page.		
Reflection *Skill Drill*	**Home Activity or Further Classroom Consolidation** Journal entry: "Summarize your activities. Include what you learned, what you discovered, and any big ideas." BLM 7.1.3		

MBF3C
BLM 7.2.1

Name:
Date:

Exponents: Take II

1. Fill in the chart

Expression to be simplified	Expanded Form	Using Exponent Laws
$\frac{2^3}{2^1}$	$\frac{2x2x2}{2}$ $= 4$	$2^{3-1} = 2^2$ $= 4$
$\frac{2^3}{2^2}$		
$\frac{2^3}{2^3}$		
$\frac{2^3}{2^4}$		
$\frac{2^3}{2^5}$		
$\frac{2^3}{2^6}$		

2. What do you notice about how the exponential expression is related to the simplified expanded expression?

3. Fill in the following Table.

Exponent Form	3^2		10^0	3^{-3}		6^0	2^{-n}		1250000^0
Simplified Form		$\frac{1}{5}$			$\frac{1}{3}$			$\frac{1}{5^m}$	

MBF3C
BLM 7.2.2

Name:
Date:

Exponents: Take 2

1. Evaluate, using a calculator. Round to 4 decimal places where necessary.

$(1.005)^{12}$=_____ 3.76^{3}=_____

5^{6}=_____ 2.15^{4}=_____

4^{6}=_____ $(17)^{0}$=_____

2^{12}=_____ $(1.045)^{6}$=_____

$(2.12)^{1}$=_____ 25^{3}=_____

1.036^{-3}=_____ 1.0075^{-12}=_____

2.16^{-4}=_____ 3^{-3}=_____

2^{-5}=_____ $(7.065)^{0}$=_____

11^{-1}=_____

2. Identify which of the expressions could have been completed without the use of a calculator. Explain.

3. Rewrite the following in simplified fraction form:

3^{-3}=_____ 2^{-5}=_____ 11^{-1}=_____

MBF3C Name:
BLM 7.2.3 Date:

Evaluate. Rewrite negative exponents and evaluate as fractions.

1. $7^0=$

2. $16^0=$

3. $-(16)^0=$

4. $(-5)^0=$

5. $-(5^0)=$

6. $5^4=$

7. $(6)^3=$

8. $7^3=$
9. $2^4=$
10. $3^3=$
11. $(-3)^3=$
12. $(-2)^4=$
13. $(-5)^3=$
14. $(-6)^4=$
15. $(-11)^1=$
16. $3^{-2}=$
17. $4^{-3}=$
18. $5^{-2}=$
19. $2^{-5}=$
20. $-(5^4)=$
21. $-(4)^{-3}=$
22. $-(3^{-2})=$
23. $-(-3)^3=$
24. $-(6)^3=$
25. $-(-2)^5=$

Unit 7 Day 3: Exponential Relations			MBF 3C
	Description Graph exponential relations, and distinguish from linear and quadratic relations		Materials Chart Paper Coloured markers TI-83s BLM 7.3.1 BLM 7.3.2
			Assessment Opportunities
Minds On…	**Whole Class →Discussion** Review what is meant by 1^{st} and 2^{nd} differences in a table of values - What is meant by $1^{st}/2^{nd}$ differences? - When did you use them before? Why? - How did you find $1^{st}/2^{nd}$ differences? Review how to graph functions on TI-83's		Y=2x $Y=2^x$ $Y=2x^2$
Action!	**Small Groups → Investigation** In groups of 3, students complete BLM 8.3.1 using TI-83's Direct students to complete their individual parts, then explain their findings to their group.		
Consolidate Debrief	**Whole Class → Discussion** On chart paper/chalkboard, summarize findings of each type of relation – Partner A's summarize together, Partner B's summarize together, Partner C's summarize together wile remaining students add to their notes. Encourage students to add to their unit cover page.		
Reflection *Skill Drill*	**Home Activity or Further Classroom Consolidation** Journal entry: "Identify and describe at least two methods for categorizing an equation as linear, exponential or quadratic." BLM 7.3.2		

MBF3C
BLM 7.3.1

Name:
Date:

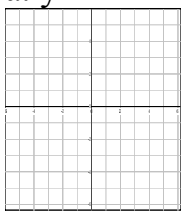
Graph-fest!

In your groups of 3, label yourselves Partner A, Partner B, and Partner C.
Each partner completes their respective part of each question.
Upon completion of Parts A, B and C, prepare to present your findings to the class.

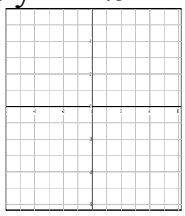
Part A

1. Using the TI-83 Graphing Calculator, graph each function:

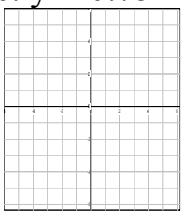
a. $y = 2x$



b. $y = -1.5x$



c. $y = 0.75x+4$



2. Sketch

Sketch

Sketch

3. Describe the graphs:

4. Complete a table of values for each:

$y = 2x$	

$y = -1.5x$	

$y = 0.75x+4$	

5. Find 1st and 2nd differences for each table of values. Describe any patterns you see.

6. Prepare to explain your findings.

MBF3C
BLM 7.3.1

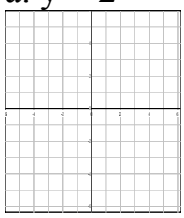
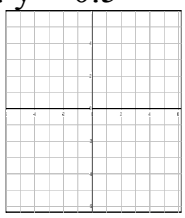
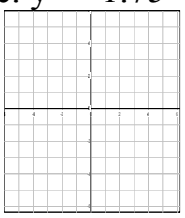
Name:
Date:

Graph-fest! (continued)

Part B

Using the TI-83 Graphing Calculator, graph each function:

a. $y = 2^x$ b. $y = 0.5^x$ c. $y = -1.75^x$

2.Sketch Sketch Sketch

3. Describe the graphs:

4. Complete a table of values for each:

$y = 2^x$

$y = 0.5^x$

$y = -1.75^x$

5. Find 1st and 2nd differences for each table of values. Describe any patterns you see.

6. Prepare to explain your findings.

MBF3C
BLM 7.3.1

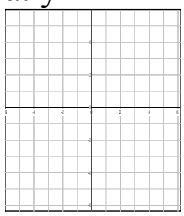
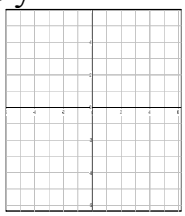
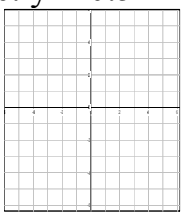
Name:
Date:

Graph-fest! (continued)

Part C

Using the TI-83 Graphing Calculator, graph each function:

a. $y = x^2$ b. $y = 2x^2$ c. $y = 0.5x^2$

2.Sketch Sketch Sketch

3. Describe the graphs:

4. Complete a table of values for each:

$y = x^2$

$y = 2x^2$

$y = 0.5x^2$

5. Find 1st and 2nd differences for each table of values. Describe any patterns you see.

6. Prepare to explain your findings.

MBF3C Name:

BLM 7.3.2 **Relations** Date:

For each question:

- Create a table of values
- Classify as Linear/Quadratic/Exponential
- Determine a formula for Linear and Exponential relations

1. James stacks cans for a grocery store display. The top row has 1 can, the second row has 2 cans, the third row has 3 cans, etc. How many cans are there in the 10^{th} row?

2. A soccer ball is kicked. At 1 sec., its height is 20.6m. At 2 sec., its height is 30.4 m. At 3 sec., its height is again at 30.4m. At 4 sec., the height is 20.6m. At 5 sec., the height is 1m.

3. A vase that cost $800 is expected to increase in value by 7% each year for 5 years.

4. Graham got a loan for a car for $16 000. After the first year, he still owed $13 171.37. After the second year, he still owed $10 167.99. After the third year, he still owed $ 6979.37. After the fourth year he still owed $3594.08. After the fifth year, he had paid off his car so there was a zero balance remaining. Use your formula to find th amount still owing after 6 months (hint: ½ or 0.5 of a year)

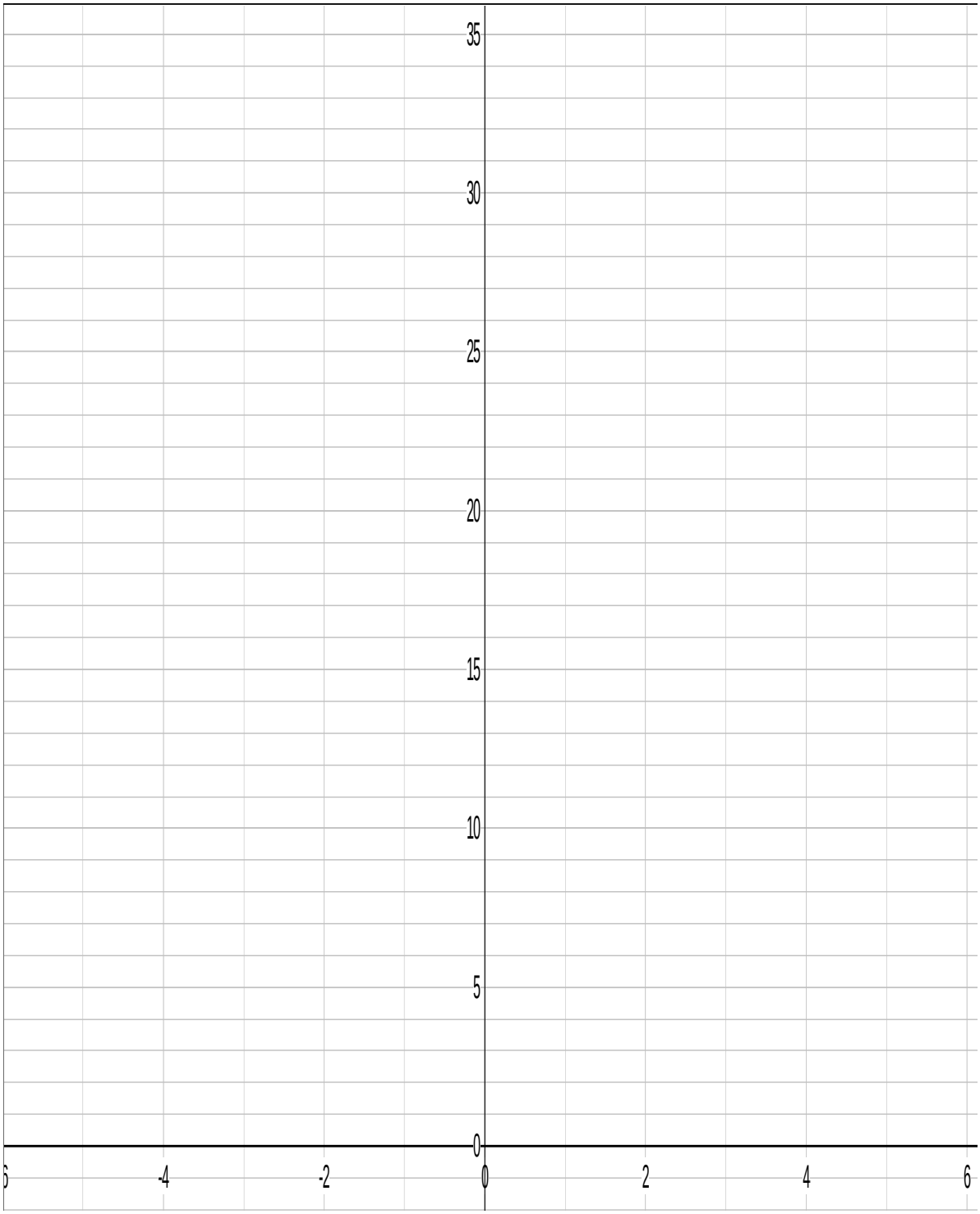
5. A $600 investment is worth $618 after 1 year, $636 after 2 years, $654 after 3 years and $672 after 4 years.

Unit 7 Day 4: Exponential Relations			MBF 3C
	Description Characteristics of Exponential Relations		Materials Grid Chart Paper or Transparencies (attached) Coloured markers or transparency markers BLM 7.4.1,7.4.2
			Assessment Opportunities
Minds On…	**Whole Class →Brainstorming** Give students the following terms, and allow 3-5 mins. Brainstorming individually, then with a partner on definitions: - x-axis - y-axis - x-intercept - y-intercept - increasing - decreasing Create definitions for each term that are agreeable to the class		If using transparencies, have students start with smallest base first, then next larger, etc. By placing transparencies on top of each other, students will see a relationship
Action!	**Small Groups →Investigation** Instruct students in small groups to complete a table of values for their equation, then graph it on grid chart paper or an overhead transparency (BLM7.4.1) Assign one graph per group $y = 1.5^x$ $y = 2^x$ $y = 2.25^x$ $y = 3.2^x$ $y = 3.45^x$ $y = 3.75^x$ $y = 4^x$ $y = 4.2^x$ $y = 4.45^x$ $y = 5^x$ $y = 5.3^x$ $y = 6^x$ Display graphs. *NOTE: Day 6 will concentrate on decreasing exponential relations. By displaying today's (increasing) relations together, then Day 6's relations together, students will continue to make connections.		
Consolidate Debrief	**Whole Class →Group Summaries** Give each group 2-3 minutes to describe their graphs, identifying x- and y-intercepts. Help students to develop the concept that each relation is increasing. Encourage students to consider things which would be represented graphically this way. Encourage students to add to their unit cover page.		
Reflection	**Home Activity or Further Classroom Consolidation** Given the equation $y = 3.5^x$, find the x- and y-intercepts. Describe the graph. Is there a value for y that is impossible? Why or why not? BLM 7.4.2		

MBF3C Name:

BLM7.4.1 Date:

TRANSPARENCY TEMPLATE/EXPONENTIAL RELATIONS I



MBF3C
BLM 7.4.2

Name:
Date:

For each exponential relation, describe its characteristics:

1. $y = 3^x$

2. $y = 2.5^x$

3. $y = 10^x$

4. $y = 1.25^x$

5. $y = 2^x$

Unit 7 Day 5: Exponential Relations			MBF 3C
	Description Exponential Relations in Real World Applications		Materials Various lengths of strips of paper BLM 7.5.1 Graph Chart paper or Transparencies (Day 4) Coloured markers
			Assessment Opportunities
Minds On…	**Whole Class →Inside/Outside Circles** Students sit in 2 circles, one inside the other. Teacher poses a question, "Person on the inside, tell the person on the outside the characteristics of the exponential relation $y=3^x$ When you are finished, say 'pass,' and then the outside person will share/extend that thinking." When finished, outside students rotate one seat to the left and continue process with the exponential relation $y=0.6^x$		
Action!	**Small Groups→Activity** Group students in 3's or 4's. Distribute 1-3 paper strips per group (each strip same length per group) Assign each group a number of cuts to make (1-4), and direct students to record this information on their sheets (BLM 7.5.1)		
Consolidate Debrief	**Whole Class →Group Presentations** Groups describe their investigation and explain/justify. Encourage students to ask questions. On last presentation of graph only, ask: What was the original amount? Each cut represents how many pieces?		
Application	**Home Activity or Further Classroom Consolidation** BLM 7.5.2		

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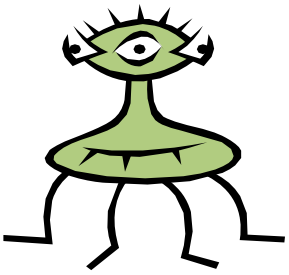
BLM 7..5.1

Name:
Date:

Germs! Germs! Germs!



Certain bacteria, under the right conditions, multiply themselves.

You will use strips of paper, each representing a bacterium, to model its growth.

For this activity, each member of your group must choose a role:

- Recorder – records data
- Counter – counts pieces for recorder
- Reader – reads questions for other group members
- Facilitator – keeps discussion of topic going

Cut your strip(s) of paper into _____ equal pieces
How any total pieces to you have?_____
Cut each piece into _____ equal pieces.
How many pieces do you have? _____
Continue to cut each piece into _____ equal pieces to complete the table:

Cuts	Pieces
0	
1	
2	
3	
4	
5	

Initial Amount

Graph your result on the transparency provided.
Identify characteristics of your graph.

- X-intercept
- Y-intercept

Create an equation to model the data.

Prepare to present your findings.

MBF3C
BLM 7.5.2

Exponential Relations

Name:
Date:

For each question, create a table of values to represent each situation then answer the questions that follow:

1. An antique costs $800. Its value increases by 15% each year.

End of Year	Value
1	$
2	$
3	$
4	$

x-intercept:

y-intercept:

Equation:

2. The population of Mathville is 23 000. Each year, the population increases at a rate of 3.4%.

End of Year	Value
1	
2	
3	
4	

x-intercept:

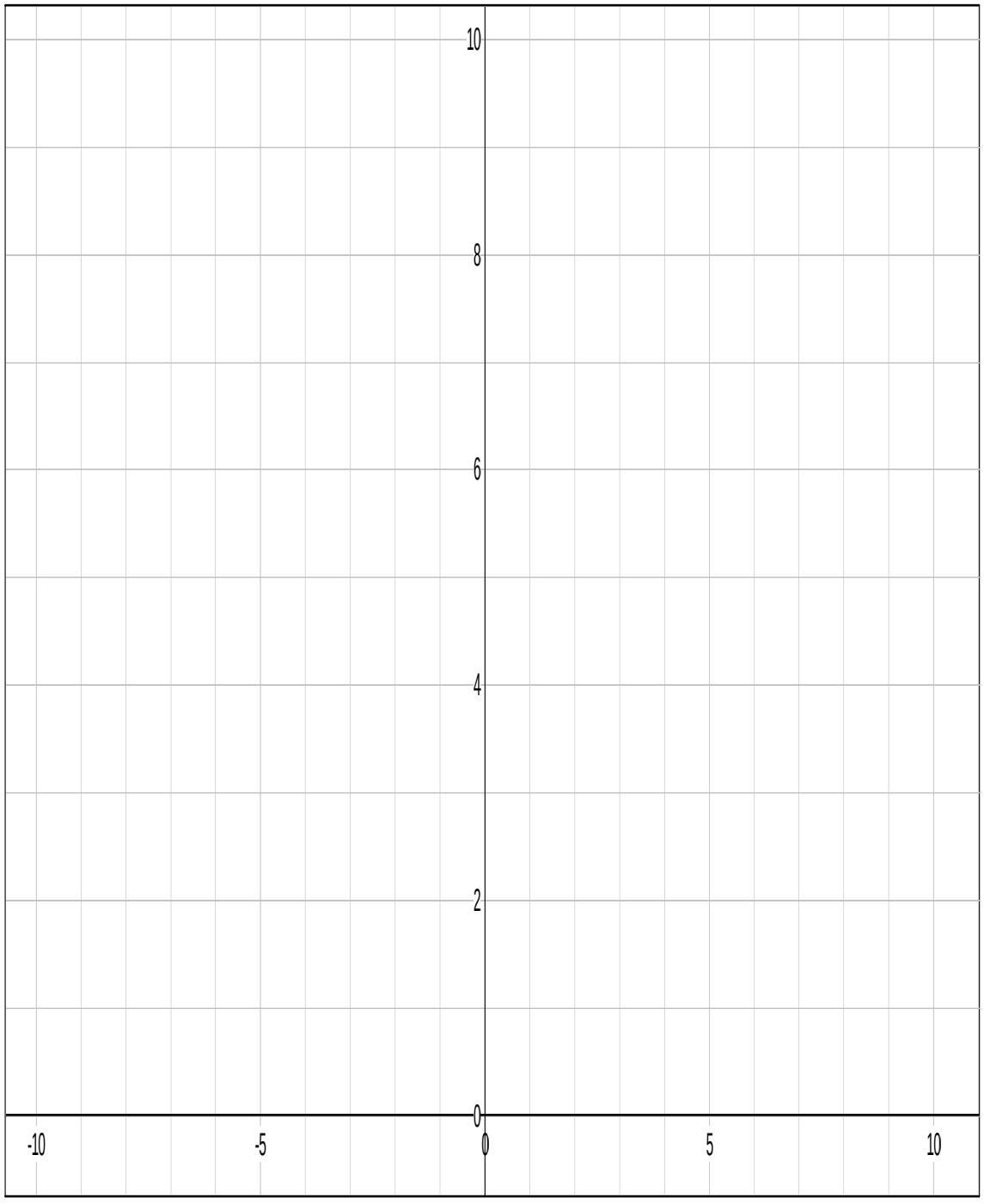
y-intercept:

Equation:

Unit 7 Day 6: Exponential Relations		MBF 3C
	Description Exponential Relations in Real World Applications	Materials BLM 7.6.1 Graph Chart paper or overhead transparency Coloured markers BLM 7.6.2

				Assessment Opportunities
	Minds On…	**Individually → Quiz** Students complete a quiz on exponents, exponent rules, and increasing exponential equations		Quiz 7.6.1
	Action!	**Small Groups → Investigation** Instruct students in small groups to complete a table of values for their equation, then graph it on grid chart paper or an overhead transparency. Assign one graph per group $y = 0.5^x$ $y = 0.12^x$ $y = 0.15^x$ $y = 0.2^x$ $y = 0.25^x$ $y = 0.3^x$ $y = 0.4^x$ $y = 0.35^x$ $y = 0.6^x$ $y = 0.7^x$ $y = 0.8^x$ $y = 0.9^x$ Display graphs. *NOTE: Display Day 6's relations together to assist students in making the connections between increasing and decreasing exponential relations.		
	Consolidate Debrief	**Whole Class → Group Summaries** Give each group 2-3 minutes to describe their graphs, identifying x- and y-intercepts. Help students to develop the concept that each relation is decreasing. Encourage students to consider things which would be represented graphically this way. Encourage students to add to their unit cover page.		
	Reflection	**Home Activity or Further Classroom Consolidation** Given the equation $y=\frac{1}{5}x$, find the x- and y-intercepts. Tell whether the graph is increasing or decreasing. Is there a value for y that is impossible? Why or why not? BLM 7.6.2		

TRANSPARENCY TEMPLATE/EXPONENTIAL RELATIONS II



MBF3C

Exponential Relations Quiz BLM 7.6.1

Name _____

Evaluate WITHOUT a calculator:

1. $2^3=$
2. $3^4=$
3. $(2^3)^2=$
4. $(-4)^3=$
5. $2^{-3}=$
6. $-(5^3)=$
7. $(-5)^3=$
8. $(5)^{-3}=$

Simplify

9. $3^5 \times 3^2=$
10. $2^{11} \div 2^6=$
11. $4^2 \times 4^{-3} \times 4=$
12. $(5^6 \times 5^2) \div 5^5=$
13. $2^6 \div 2^3=$
14. $(3^6 \div 3^2) \div 3^4=$
15. $(-1335)^0=$

For each of the following increasing exponential functions, place them in order from steepest to least steep.

$Y = 3^x$

$y = 2.5^x$

$y = 10^x$

$y = 1.25^x$

$y = 2^x$

MBF3C
BLM 7.6.2

Name:
Date:

Exponential Relations

For each exponential relation, describe its characteristics:

1. $y = 0.75^x$

2. $y = 0.35^x$

3. $y = 0.2^x$

4. $y = 0.15^x$

5. $y = 0.8^x$

Unit 7 Day 7: Exponential Relations			MBF 3C
	Description Exponential Relations in Real World Applications		Materials BLM 7.7.1 Number cubes (100 per group) Graph paper
			Assessment Opportunities
Minds On…	**Whole Class →Discussion** Review/Discuss previous lesson's summary of exponential equations. Return and take up quizzes. Take up Worksheet from yesterday. Using transparencies of exponential functions from previous lessons, help students to make connections to when a function is increasing/decreasing, the initial amount, and how that applies to the equation		
Action!	**Small Groups →Investigation** Small groups of 3-4 students use 100 number cubes to complete the investigation. Roles: Partner A is Roller, Partner B is Recorder of data and Partner C is Summarizer. Partner D (if necessary) is Facilitator/Questioner. Ask the students if they feel the decay rate was well modelled by the dice, and if the numbers were reasonable.		
Consolidate Debrief	**Whole Class → Student Summaries** Encourage communication using oral summaries of activity. Encourage students to add key ideas to unit cover page.		
Application	**Home Activity or Further Classroom Consolidation** Students complete Part D of BLM 7.7.1		

MBF3C Name:
BLM 7.7.1 Date:

Half-Life of Radioactive Materials

Radioactive materials break down in a process known as *radioactive decay*. The rate of decay varies from substance to substance.
The *TIME* it takes for half (½) of a substance to decay is known as *half-life*.
e.g. If the half-life of a substance is 20 minutes, then a 6kg mass will decay to 3 kg in 20 minutes.

Procedure: In this investigation, you will model the decay of radioactive iodine over several days using 100 numbered cubes to represent 100 atoms of iodine.

Part A: Data Collection

1. If approximately 1/6 of the atoms decay each day, then each "pour" of the atoms represents one day. Atoms with the marking of "one" facing UP will decay each day.
2. Repeat for several days, recording the number of atoms that remain after each "day of radioactive decay" until there are less than five (5) atoms remaining.

Part B: Data Presentation
Represent your findings by constructing a graph of Number of Days versus Number of Atoms Remaining on the graph paper provided.

Part C: Conclusions

1. Based on your observations, numerically and graphically, predict the half-life of iodine. Explain your reasoning.

2. Will the sample of iodine ever completely decay to zero atoms? Explain your answer.

Half-Life of Radioactive Materials (continued)

3. Write an equation to represent the number of atoms remaining as a function of time.

Part D: Extension

Create an activity to introduce and model half-life using two coloured counters. Be sure to include the solution with your activity.

Unit 7 Day 8: Exponential Relations			MBF 3C
	Description Exponential Relations in Real World Applications		Materials BLM 7.8.1 Graph paper Calculators
			Assessment Opportunities
Minds On…	**Whole Class →Discussion** Review/Discuss previous lesson. Discuss student activities presented in Day 7's Part D.		
Action!	**Small Groups→Brainstorming** Small groups of 3-4 students brainstorm on BLM 7.8.1, no writing utensils in hand. **Individually →Activity** Each student completes the Cat and Mouse activity. They may use calculators, graph paper		
Consolidate Debrief	**Whole Class →Discussion** After collecting the activity, discuss approaches to solutions and methods used.		
Concept Practice	**Home Activity or Further Classroom Consolidation** Review of Exponential Relations BLM 7.8.2		

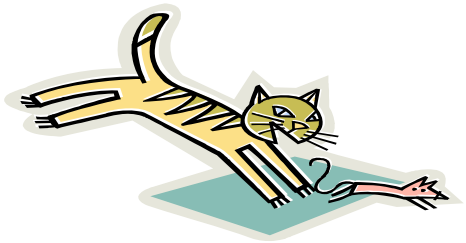
MBF3C
BLM 7.8.1

Name:
Date:

Cats and Mice!

There is an isolated island off the West coast of Canada. The island has become overrun with mice, so the Wildlife Federation of Canada released a cat population on the island to stabilize the mouse population. In 1999, the population of the mice was 23,576 and began to decrease at a rate of 2.5% per year. In the same year, the population of cats was at 15,786 and was increasing at a rate of 1.8% per year. Assume that there is no outside factor, and that these rates continue in order to answer the following questions.

1. Create a table of values for each population. Find AND analyze the first-differences. What can you say about the population-growth/decay?
2. Create an exponential function that describes the population of the mice AND create an exponential function that describes the population of the cats. How did you come up with this equation?
3. On the same graph, plot the function that represents the population of the mice AND the function that represents the population of the cats.
4. How do the populations differ? How are they related?
5. When would the population of the cats be greater than the population of the mice?
6. When would the populations be the same? How can you tell?
7. What will happen to both the mice and cats populations if this trend continues?
8. Write a brief paragraph summarizing your findings regarding the mice and cats populations.



MBF3C Name:

BLM 7.8.2 **Exponential Relations** Date:

Evaluate:

1. $4^6 \div 4^3 =$
2. 16^0
3. 11^{-1}
4. $4^5 \times 4^{-2}$
5. $(3^2)^3$
6. 2^5
7. $(2^4)^2$
8. $5^8 \div 5^4$
9. $5^2 \times 5^2$
10. $3^8 \div 3^5$
11. 5^{-3}
12. $(4^3)^2$

Identify each of the following equations as either linear, exponential or quadratic.

13. $y = 3x$
14. $y = 3^x$
15. $y = 3x^2$
16. $y = -0.75x$
17. $y = -0.75^x$
18. $y = -0.75x^2 + 2$
19. $y = x^2 + 5$
20. $y = 16^x$

For each exponential situation, identify its characteristics:

21. A club uses email to contact its members. The chain starts with 3 members who each contact three more members. Then those members (9) each contact 3 members, and so the contacts continue.

22. A bouncing ball rebounds to 0.75 of its height on each bounce. The ball was dropped from a height of 30 metres.

23. A painting was bought for $475. Each year, its value increases by 8%.

MBF3C Unit 7 Solutions

BLM7.1.1

PART A:

1.a. 9 x 27 = 243; ¼ x 1/8 = 1/32; 0.00001 x 0.001 = 0.00000001

b. 8 x 32 = 256; 1/9 x 1/3 = 1/27; 0.008 x 0.0016 = 0.0000128

c. 25 x 625 = 15625; 1/64 x 1/16 = 1/1024; 0.3 x 0.00243 = 0.000729

(answers will vary)Each pair of answers is the same. It looks like the exponents get added.

2.a. 256÷32=8; 1/64÷1/4=1/16; 0.0000128÷0.008=0.0016

b. 15625÷25=625; 1/32÷1/8=1/4; 0.000729÷0.00243=0.3

c. 243÷9=27; 1/243÷1/27=1/9; 0.00000001÷0.00001=0.001

(answers will vary)Each pair of answers is the same. It looks like the exponents get subtracted.

3.a. $(2^3)^2=(8)^2=64$; $(.3^2)^2=(0.09)^2=0.0081$; $(½^4)^1=(1/16)^1=1/16$

b. $(3^2)^3=(9)^3=729$; $(5^3)^2=(125)^2=15625$; $(⅓^4)^2=(1/81)^2=1/6561$

c. $(.1^3)^2=(0.001)^2=0.000001$; $(2^2)^4=(4)^4=256$; $(¼^2)^3=(1/16)^3=1/4096$

(answers will vary) Each pair of answers is the same. It looks like the exponents get multiplied.

PART B: (ANSWERS WILL VARY)

1. *students ought to develop the laws of exponents:*
2. $m^a \times m^b = m^{a+b}$; $m^a \div m^b = m^{a-b}$; $(m^a)^b=m^{ab}$

BLM7.1.2

1. 15625
2. 4
3. 1/243
4. 64
5. 4
6. 46656
7. 343
8. 25
9. 16
10. 1/27
11. 8
12. 1
13. 7
14. 1000
15. 279 936
16. 1/8
17. 4096
18. 729
19. 1024
20. 1/15625
21. 1/4096
22. 4096
23. 729
24. 1024
25. 1/512
26. 3^{a+b+c}
27. 4^{2a}

MBF3C Unit 7 Solutions

BLM7.2.1

2. that they are equal

BLM7.2.2

1. 1.0617; 53.1574; 15625; 21.3675; 4096; 1; 4096; 1.3023; 2.12; 15625; 0.8993; 0.9142; 0.0459; 0.0370; 0.03125; 1; 0.0909
2. (answers will vary): anything with a zero or 1 exponent could have been done without a calculator because of the exponent rules. Also, negative exponents (smaller digits) like 3^{-3} can be simplified in fraction form.
3. 1/27; 1/32; 1/11

BLM7.2.3

1. 1
2. 1
3. -1
4. 1
5. -1
6. 625
7. 216
8. 343
9. 16
10. 27
11. -27
12. 16
13. -125
14. 1296
15. -11
16. 1/9 or 0.11111
17. 1/64 or 0.015625
18. 1/25 or 0.04
19. 1/32 or 0.03125
20. -625
21. – 1/64 or -0.015625
22. -1/9 or -0.1111
23. 27
24. -216
25. 32

MBF3C Unit 7 Solutions

BLM7.3.1

PART A:

3. (answers will vary) They are all lines
4. (answers will vary)

y = 2x	
-2	-4
-1	-2
0	0
1	2
2	4
3	6

y = -1.5x	
-2	3
-1	1.5
0	0
1	-1.5
2	-3
3	-4.5

y = 0.75x+4	
-2	2.5
-1	3.25
0	4
1	4.75
2	5.5
3	6.25

5. First Differences: (a) 2; (b) -1.5; (c) 0.75
   Second Differences: (a) 0 (b) 0; (c) 0
   (answers will vary) The first differences are in the equation (slope), second differences are all zero.

PART B:

3(answers will vary) The graphs are curves/exponential graphs

4. (answers will vary)

$y = 2^x$	
-2	0.25
-1	0.5
0	1
1	2
2	4
3	8

$y = 0.5^x$	
-2	4
-1	2
0	1
1	0.5
2	0.25
3	0.125

$y = -1.75^x$	
-2	-0.3265
-1	-0.5714
0	-1
1	-1.75
2	-3.0625
3	-16.413

5. First Differences: (a) 0.25, 0.5, 1, 2, 4; (b) -2, -1, -0.5, -0.25, -0.125; (c) -0.2449, -0.4286, -0.75, -1.3125, -13.3505
   Second Differences: (a) 0.25, 0.5, 1, 2; (b) 1; 0.5, 0.25, 0.375
   (answers may vary) Don't really notice anything

MBF3C Unit 7 Solutions

PART C:

3. (answers may vary) They are all parabolas
4. (answers may vary)

$y = x^2$

-2	4
-1	1
0	0
1	1
2	4
3	9

$y = 2x^2$

-2	8
-1	2
0	0
1	2
2	8
3	18

$y = 0.5x^2$

-2	2
-1	0.5
0	0
1	0.5
2	2
3	4.5

5. First differences: (a) -3, -1, 1, 3, 5; (b) -6, -2, 2, 6, 10; (c) -1.5, -0.5, 0.5, 1.5, 2.5
   Second Differences: (a) 2, 2, 2, 2 (b) 4, 4, 4, 4; (c) 1, 1, 1, 1
   (answers will vary) Second differences are the same

BLM7.3.2

(Table of values will vary for each question)

1.

X	Y
1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	8
9	9
10	10

The function is linear because the first differences are the same.

Equation: y=x

MBF3C Unit 7 Solutions

2.

X	Y
1	20.6
2	30.4
3	30.4
4	20.6
5	1

The function is quadratic because the second differences are the same

Equation: VARIES

3.

X	Y
0	800
1	856
2	915.92
3	980.03
4	1048.64
5	1122.04

The function is exponential because it is neither linear nor quadratic (first and second differences reveal no patterns).

Equation: $y=800(1.07)^x$

4.

X	Y
0	16 000
1	13 171.37
2	10 167.99
3	6979.37
4	3594.08
5	0

The function is exponential because it is neither linear nor quadratic (first and second differences reveal no patterns).

Equation: VARIES

5.

X	Y
0	600
1	618
2	636
3	654
4	672

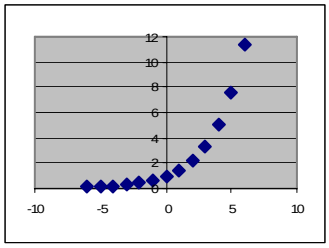
The function is linear because first differences are the same.

Equation: y=18x+600

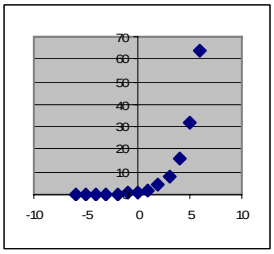
MBF3C Unit 7 Solutions

DAY 4 CLASS GRAPHS:

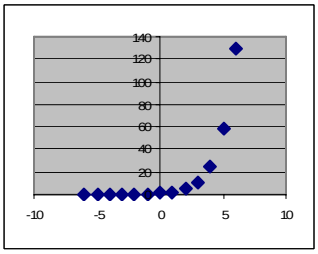
$y = 1.5^x$



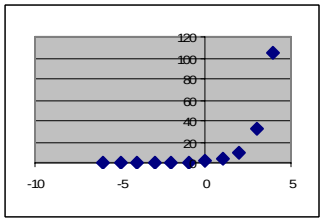
$y = 2^x$



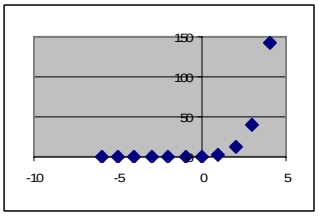
$y = 2.25^x$



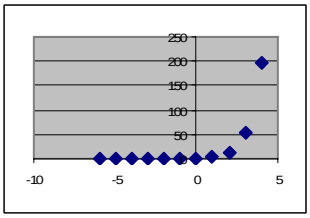
$y = 3.2^x$



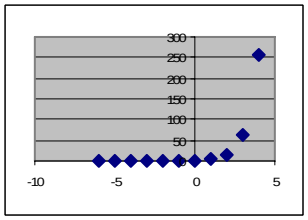
$y = 3.45^x$



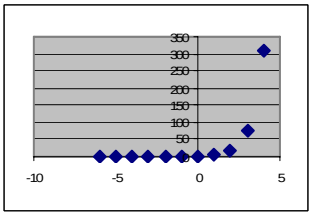
$y = 3.75^x$



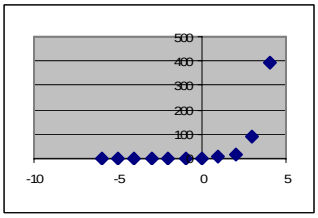
$y = 4^x$



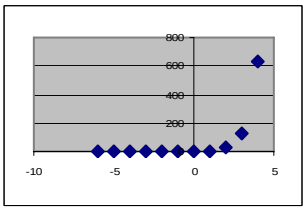
$y = 4.2^x$



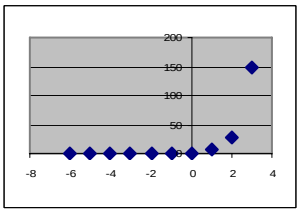
$y = 4.45^x$



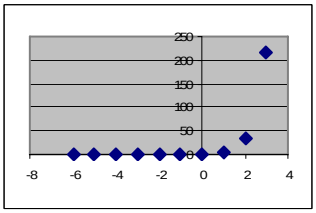
$y = 5^x$



$y = 5.3^x$



$y = 6^x$



MBF3C Unit 7 Solutions

Worksheet 8D4

1. x-intercept: none
   y-intercept: 3
   increasing
2. x-intercept: none
   y-intercept: 2.5
   increasing
3. x-intercept: none
   y-intercept: 10
   increasing
4. x-intercept: none
   y-intercept: 1.25
   increasing
5. x-intercept: none
   y-intercept: 2
   increasing

BLM7.5.1

Answers will vary according to the number of cuts assigned to each group, as well as the number of strips assigned to each group

Worksheet 8D5

1.

End of Year	Value
1	$920
2	$1058
3	$1216.70
4	$1399.21

x-intercept: none

y-intercept: 800

Equation: $y=800(1.15)^x$

2.

End of Year	Value
1	23782
2	24590.588
3	25426.66799
4	26291.17

x-intercept: none

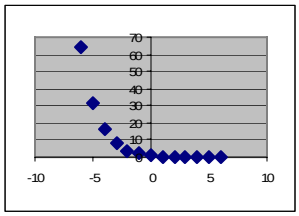
y-intercept: 23 000

Equation: $y = 23\ 000\ (1.034)^x$

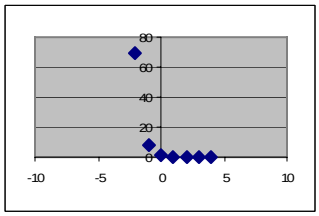
MBF3C Unit 7 Solutions

DAY 6 CLASS GRAPHS

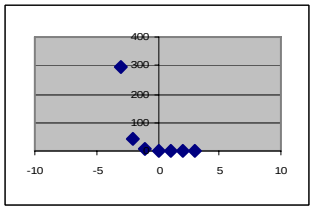
$y = 0.5^x$



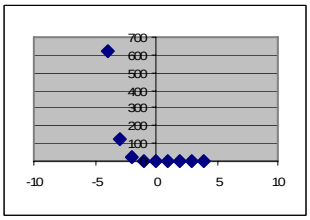
$y = 0.12^x$



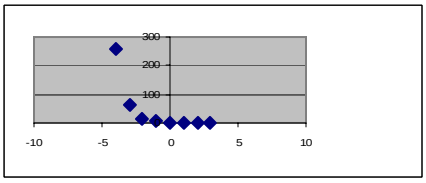
$y = 0.15^x$



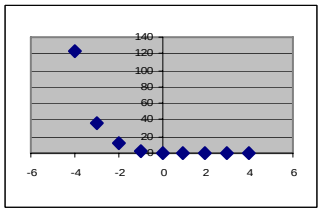
$y = 0.2^x$



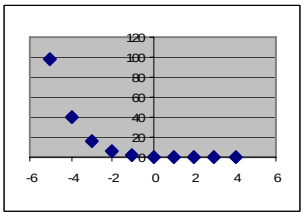
$y = 0.25^x$



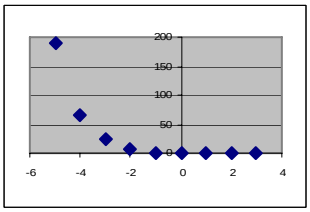
$y = 0.3^x$



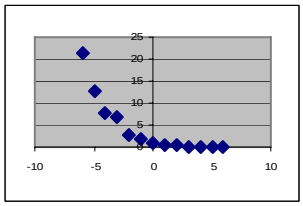
$y = 0.4^x$



$y = 0.35^x$



$y = 0.6^x$



WORKSHEET 8D6

1. x-intercept: none
   y-intercept: 0.75
   decreasing
2. x-intercept: none
   y-intercept: 0.35
   decreasing
3. x-intercept: none
   y-intercept: 0.2
   decreasing
4. x-intercept: none
   y-intercept: 0.15
   decreasing
5. x-intercept: none
   y-intercept: 0.8
   decreasing

MBF3C Unit 7 Solutions

EXPONENTIAL RELATIONS QUIZ BLM 7.6.1

1. 8
2. 81
3. 64
4. -64
5. 1/8
6. -125
7. -125
8. 1/125
9. 3^7
10. 2^5
11. 1
12. 5^3
13. 2^3
14. 1
15. 1

$y = 10^x$ is steepest
$Y = 3^x$
$y = 2.5^x$
$y = 2^x$
$y = 1.25^x$ is least steep

BLM7.7.1

(answers will vary)
PART C:
It will take between 3 and 4 days for half of a sample of iodine to decay (half-life)
The amount of the sample will never reach zero because there will always be 5/6 of the sample left. So even with a small sample, an even smaller sample will be left.
$Y=100(5/6)^x$
PART D: (answers will vary) One roll or flip is a half-life

MBF3C Unit 7 Solutions

BLM7.8.1

1. (answers will vary)

Mice

Yr	Pop.
1999	23 576
2000	22 986.6
2001	22 411.9
2002	21 851.6
2003	21 305.3
2004	20 772.7
2005	20 253.4
2006	19 747.1
2007	19 253.4

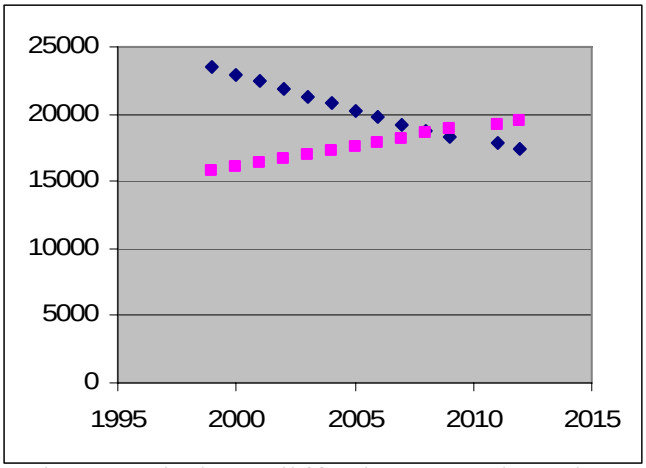
Cats

Yr.	Pop.
1999	15 786
2000	16 060.1
2001	16 359.4
2002	16 653.9
2003	16 953.6
2004	17 258.8
2005	17 569.5
2006	17 885.7
2007	18 207.7

Mice first differences do not reveal any pattern
Mice second differences do not reveal any pattern
Cats first differences do not reveal any pattern
Cats second differences do not reveal any pattern
Mice population is decreasing. Cat population is increasing.

2. Mice: y=23 576 (0.975)x because 23576 is the initial population, while it decreases at a rate of 0.975.
Cats: y=15 786(1.018)x because 15786 is the initial population, while it increases at a rate of 1.018

3.



4. The populations differ because the mice population decreases, while the cat population increases. They are related because the cat population is the large cause for the decrease in the mice population.

MBF3C Unit 7 Solutions

5. The population of the cats will be greater than the population of the mice in 2009.
6. The populations would be the same sometime between 2008 and 2009 because in 2008 the mice population (18772) will be greater than the cat population (18535.4), but in 2009 the mice population (18302.7) will be less than the cat population (18869).
7. Eventually the mice population will decline to less than one (zero), and the cat population will continue to increase.
8. ANSWERS WILL VARY

WORKSHEET 8D8

1. $4^3=64$	2. 1	3. 1/11
4. $4^3=64$	5. $3^6=729$	6. 32
7. $2^8=256$	8. $5^4=625$	9. $5^3=125$
10. $3^3=27$	11. 1/125	12. $4^6=4096$

13. Linear
14. Exponential
15. Quadratic
16. Linear
17. Exponential
18. Quadratic
19. Quadratic
20. Exponential
21. x-intercept: none, y-intercept: 3, increasing
22. x-intercept: none, y-intercept: 30, decreasing
23. x-intercept: none, y-intercept: 475, increasing